An Roinn Oideachais
agus Scileanna

Applied Mathematics

Curriculum Specification

LEAVING CERTIFICATE
Ordinary and Higher Level

Contents

1 Senior cycle

Senior cycle students are approaching the end of their time in school and are focusing on the directions they would like to take in their future lives. Senior cycle plays a vital role in helping students to address their current needs as young adults and in preparing them for life in a changing economic and social context.

Senior cycle is founded on a commitment to educational achievement of the highest standard for all students, commensurate with their individual abilities. To support students as they shape their own future there is an emphasis on the development of knowledge and deep understanding; on students taking responsibility for their own learning; on the acquisition of key skills; and on the processes of learning. The broad curriculum, with some opportunities for specialisation, supports continuity from junior cycle and sets out to meet the needs of students, some of whom have special educational needs, but who all share a wide range of learning interests, aptitudes and talents.

Subjects at senior cycle promote a balance between knowledge and skills, and the kinds of learning strategies relevant to participation in, and contribution to, a changing world where the future is uncertain.

The experience of senior cycle

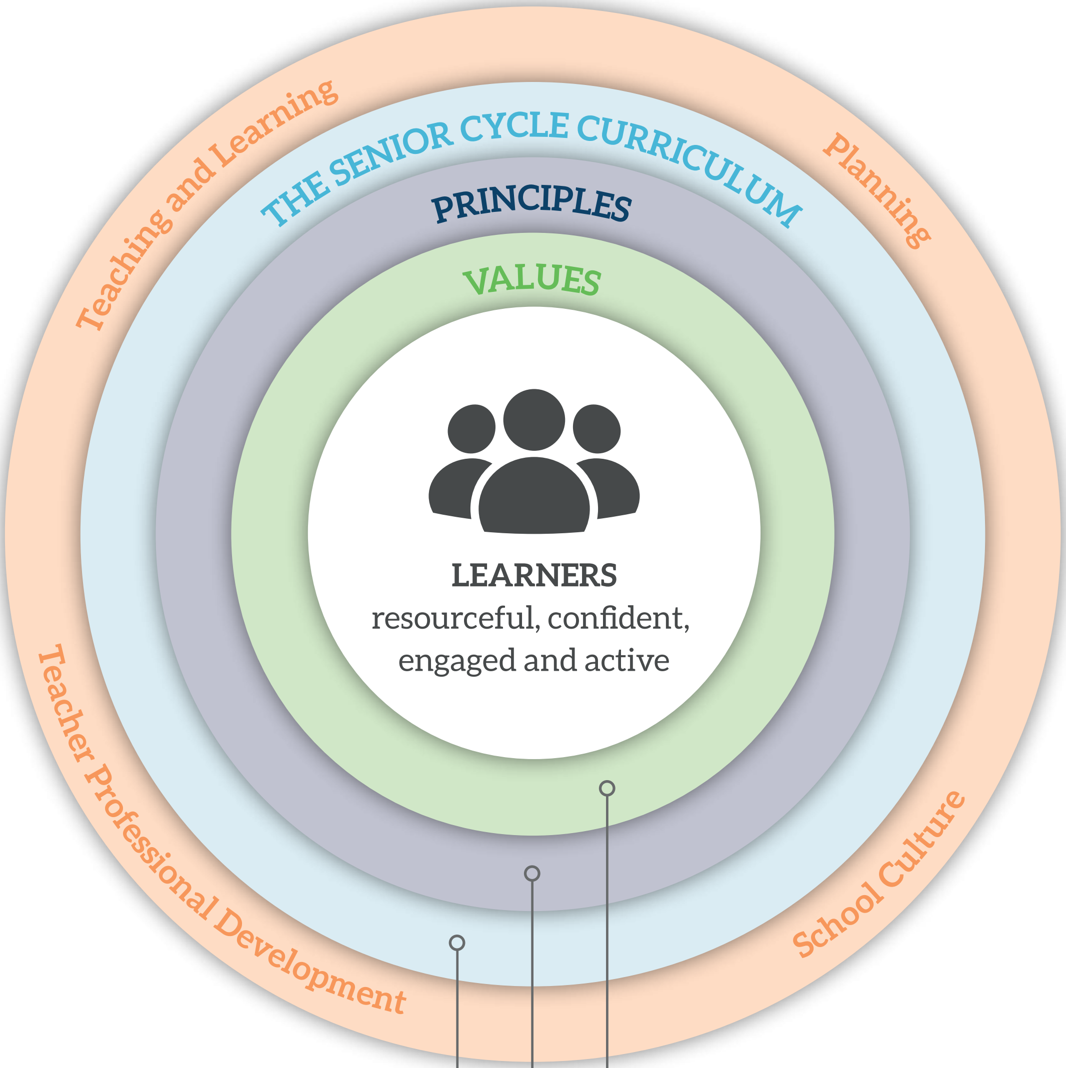
The vision of senior cycle sees the learner at the centre of the educational experience. That experience will enable students to be resourceful, to be confident, to participate actively in society, to build an interest in learning, and to develop an ability to learn throughout their lives.

This vision of the learner is underpinned by the values on which senior cycle is based and it is realised through the principles that inform the curriculum as it is experienced by students in schools. The specification has embedded key skills, clearly expressed learning outcomes, and is supported by a range of approaches to assessment; it is the vehicle through which the vision becomes a reality for the learner.

At a practical level, the provision of a high-quality educational experience in senior cycle is supported by:

- effective curriculum planning, development, organisation and evaluation
- teaching and learning approaches that motivate and interest students, that enable them to progress, that deepen and apply their learning, and that develop their capacity to reflect on their learning
- professional development for teachers and school management that enables them to lead curriculum development and change in their schools
- a school culture that respects students, that encourages them to take responsibility for their own learning over time, and that promotes a love of learning.

Senior cycle education is situated in the context of a broader education policy that focuses on the contribution that education can make to the development of the learner as a person and as a citizen. It is an education policy that emphasises the promotion of social cohesion, the growth of society and the economy, and the principle of sustainability in all aspects of development.



THE SENIOR CYCLE CURRICULUM

learning outcomes

key skills

subjects

short courses

transition units

assessment

certification

guidance

PRINCIPLES

quality

inclusive education

continuity

choice and flexibility

participation, relevance and enjoyment

well-being

creativity and innovation

lifelong learning

VALUES

human dignity and respect

equality and inclusion

justice and fairness

freedom and democracy

Figure 1: Overview of senior cycle

RESOURCEFUL

they show their imagination, intelligence, intuition and other talents through

curiosity

enquiry

open-mindedness

reflection

connecting learning

innovation

problem solving

creativity

CONFIDENT

they develop their physical and mental well-being and

become self-aware

have high self-efficacy

engage with ethics, values and beliefs

welcome opportunities

can cope with setbacks

can effect positive change

LEARNERS
resourceful, confident, engaged and active

ENGAGED

they participate in the social, community, national and international dimensions of their lives by

showing respect for others

forming and sustaining caring relationships

making informed decisions

building practical know-how

taking interest in and responsibility for their social and physical environment

developing moral/ethical and political understanding

making lifestyle choices that are sustainable

contributing to their own material wellbeing and the material wellbeing of society

ACTIVE LEARNERS

they pursue excellence in learning to the best of their ability and develop a love of learning by

seeking and using knowledge, and understanding how knowledge is created

experiencing passion for, rigour in and commitment to learning

developing intellectual and critical thinking skills

exercising autonomy and independence in learning

managing their learning and making learning choices

setting and achieving learning goals

pursuing learning qualifications

Figure 2: The vision of the learner.

2 Leaving Certificate Applied Mathematics

Introduction

Applied mathematics is the use of the language of mathematics to study and solve real-world problems. It is multi-disciplinary in nature; applied mathematicians collaborate with colleagues in many disciplines using quantitative techniques and high-performance computing to shed light on complex problems in their field.

Mathematical modelling is the process through which applied mathematicians use mathematics to represent, analyse, make predictions and provide insight into real-world phenomena. Used in a variety of scientific disciplines such as theoretical physics, bioinformatics, robotics, image processing, chemistry, economics, engineering, and finance, models are abstractions of reality that respect reality. These abstractions can lead to scientific advances, be the foundation for new discoveries, and help leaders make informed decisions.

Mathematical models are ubiquitous, providing a quantitative framework for understanding, predicting and making decisions in nearly every aspect of our lives. Whether it is weather forecasting, climate research, population dynamics, or exploring distant planets, without algorithms and tools from mathematical models, none would work as efficiently and accurately as they do.

Leaving Certificate Applied Mathematics introduces modelling through exploration of real problems in the physical, natural, and economic worlds. Modelling requires students to turn authentic situations into mathematical structures. They then operate on those mathematical structures and generate a solution or a strategy to address the situation. The cycle of defining the problem, translating it to mathematics, calculating and evaluating the solution provides some of the most challenging, exhilarating, democratic work students will ever do in mathematics, requiring the best from all of them, even the ones who dislike mathematics.

Aim

Leaving Certificate Applied Mathematics aims to develop the learner's capacity to use mathematics to model real-world problems. By focusing on all aspects of the problem-solving cycle it is envisaged that learners will move beyond calculating procedures and gain experience in asking appropriate questions, formulating mathematical representations of problems, and interpreting and verifying results. Through Applied Mathematics, students should learn to appreciate the extent to which mathematics is relevant in everyday life, generating engagement and interest in the process. It is anticipated that digital technology will be used as a learning tool in some aspects of this course.

Objectives

The objectives of Leaving Certificate Applied Mathematics are to develop applied mathematical problem-solving skills so that students will be able to:

- Formulate a problem: Consider the scope and detail of a real-world problem, and to define manageable questions to address
- Translate the problem into mathematics: Create or choose a suitable mathematical model, and then formulate the question as a mathematical problem within the model
- Compute a solution: Use mathematical techniques to solve the mathematical problem
- Evaluate the solution: Interpret the mathematical solution in the original context.

Related learning

Leaving Certificate Applied Mathematics builds on the knowledge, attitudes and a broad range of transferable skills that stem from a learner's early childhood education, through primary school and the junior cycle curriculum.

EARLY CHILDHOOD

Aistear, the early childhood curriculum framework, celebrates early childhood as a time of wellbeing and enjoyment where children learn from experiences as they unfold. The theme of *Exploring and Thinking* is about children making sense of the things, places and people in their world by interacting with others, playing, investigating, questioning, and forming, testing and refining ideas. Children's interests and play should be the source of their first mathematical experiences. Experiences become mathematical by being represented in another form. Young children represent their ideas by talking, but also through models and graphics. From the motoric and sing-song beginnings of rhymes stem the geometric patterns built from unit blocks and the gradual generalisation and abstraction of patterns experienced throughout the child's day. The theme of *Communicating* is about children sharing these experiences, thoughts, ideas, and feelings with others with growing confidence and competence in a variety of ways and for a variety of purposes.

PRIMARY EDUCATION

The mathematics curriculum at primary school aims to provide children with a language and a system through which to analyse, describe, illustrate and explain a wide range of experiences, make predictions, and solve problems. Mathematics education seeks to enable students to think and communicate quantitatively and spatially, solve problems, recognise situations where mathematics can be applied, and use appropriate technology to support such applications.

The integrated programme of Social, Environmental and Scientific Education (SESE) in primary schools provides opportunities for children to actively explore and investigate the world around them from a human, social and cultural perspective. A scientific approach to investigations fosters the development of important skills, concepts and knowledge through which children can observe, question, investigate, understand and think logically about living things and their environments, and about materials, forces, everyday events and problems. The knowledge and skills acquired may be applied in designing and making activities in which children perceive a need to create or modify elements of their environments. Through their investigations, children develop informed, critical and scientific perspectives that acknowledge the importance of founding judgements on a respect for facts, accuracy and reason. The Designing and making focus of the SESE curriculum fosters the skills of exploring, planning, designing and making, and enables children to apply their scientific knowledge and understanding to devising a method or solution, carrying it out practically and evaluating the final product. As these skills are developed progressively through the primary school, children will build a solid foundation for analysing open-ended problem-solving tasks and developing computational thinking skills in Leaving Certificate Applied Mathematics.

A number of the core strands of the English curriculum at primary school—Competence and confidence in using language, Developing cognitive abilities through language, and Emotional and imaginative development through language—contribute to important aspects of a child's development and to the development of skills that will be relevant for Leaving Certificate Applied Mathematics.

JUNIOR CYCLE

Many of the junior cycle Statements of Learning relate strongly to Leaving Certificate Applied Mathematics, especially those that focus on problem-solving, design, communication skills, and understanding the role and contributions of technology in society. In addition, all the key skills required for successful learning by students across the curriculum at junior cycle are relevant for Leaving Certificate Applied Mathematics:

Many junior cycle subjects and short courses have close links with Leaving Certificate Applied Mathematics, particularly mathematics, science, business studies, and the short course in coding.

SENIOR CYCLE

Applied Mathematics is inherently a transdisciplinary subject, authentic and relevant to the real world. Transdisciplinary learning is not confined by traditional subjects but is supported and enriched by them. The knowledge and understanding gained in Applied Mathematics can be used in conjunction with that developed in other senior cycle subjects, including mathematics, computer science, engineering, physics, chemistry, biology, agricultural science, business and economics, to enrich overall learning. Students will appreciate the power of mathematics to model and shed light on complex problems in many discipline areas as well as in more complex real-life situations they encounter in their lives.

FURTHER STUDY

Applied Mathematics builds a solid foundation for students who wish to move on to further study and training in specialised areas such as engineering, technology-related jobs, computer science, education, mathematics, science, business and finance. In addition, applied mathematics incorporates a broad range of transferable problem-solving skills and techniques, including logical thinking, creative design, synthesis, and evaluation. It teaches a range of generically useful skills in areas such as communication, time management, organisation, and teamwork. These skills are relevant to all further study, and indeed all learning beyond formal education.

Applied Mathematics will help young people to use mathematics to better understand the world they live in; it will provide them with confidence to use data to draw defensible conclusions and use their mathematical knowledge and skills to make real-life impact.

COMMUNITY AND SOCIETY

Many important aspects of life in society are being transformed through mathematical modelling and applications of mathematical models. Therefore, the development of an expert as well as a layperson competence in the general population to critique mathematical models and the ways in which they are used in decision-making, is becoming imperative for developing and maintaining societies based on equality and democracy.

3 Overview and structure

There are four strands in Leaving Certificate Applied Mathematics: Mathematical modelling, Mathematical modelling with networks and graphs, Mathematically modelling the physical world, and Mathematically modelling a changing world.

Mathematical modelling underpins applied mathematics and so is considered a unifying strand (Figure 3); it permeates all the strands of the specification. The learning outcomes in this strand identify the knowledge, skills and values which are essential to students' learning throughout the course, underpinning the activities and content in the other strands.

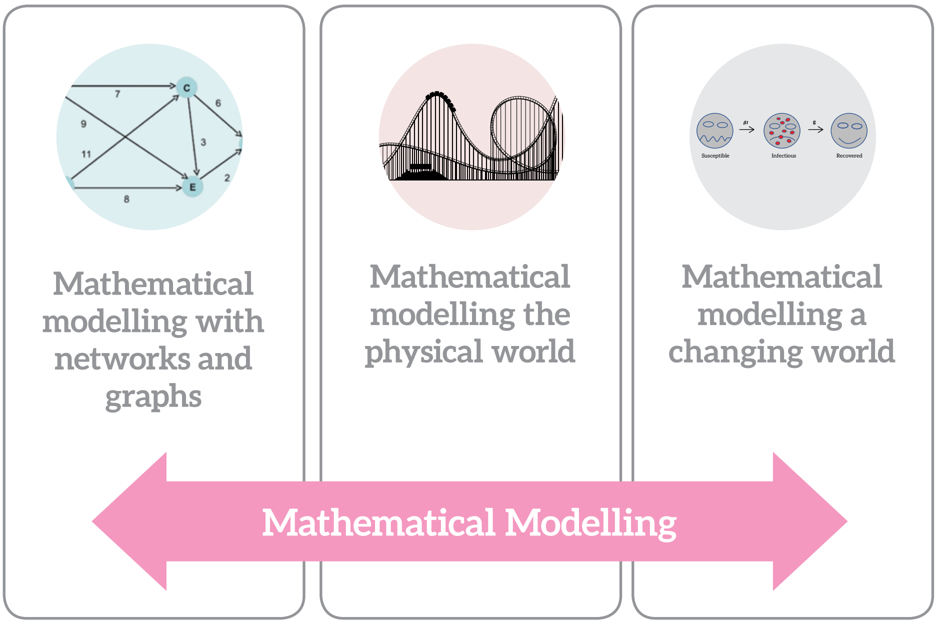


Figure 3: Overview of the specification.

The Applied Mathematics course uses authentic problems as a context for learning about the application of mathematics to design solutions for real-world situations. Problems cross disciplines. For example, from an engineering context: can we accelerate a car to 1000kph? In a business context: where do you build distribution centres to minimise a company's transportation costs? From a medical context: how can we minimise the spread of a disease such as SARS? Students use and design mathematical models to address the authentic problem from the real world and use numerical methods to solve it, whilst at the same time grasping the essence of the real-world context and the mathematics that they used to solve the problem.

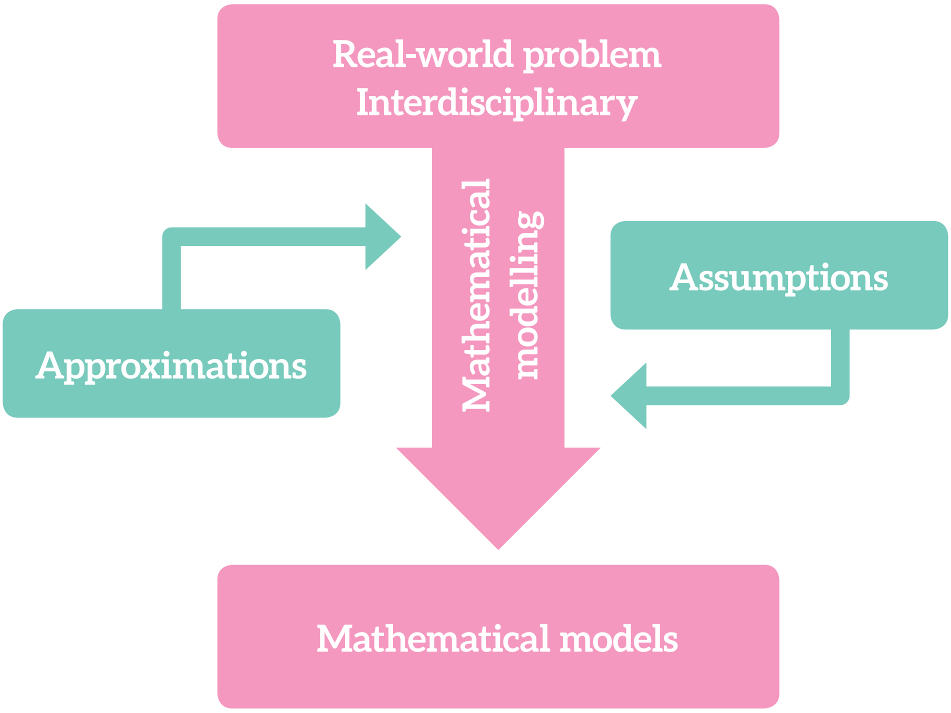


Figure 4: Mathematical modelling.

The modelling process (Figure 4) is iterative and involves making assumptions and approximations. If the approximations are good enough, applied mathematicians can get a useful solution to the real-world problem that is of benefit to the community. Students learn how to complete the full modelling process in strand 1.

In strands 2, 3 and 4, students will encounter both modelling and word[1] problems that cross many disciplines. As they model these real-world problems and engage with techniques and algorithms they will learn to abstract and to apply analytical techniques in a wide range of application areas. Students will see the power of mathematics as they build and use mathematical models to solve authentic problems relevant to their lives. It is expected that, in this course, students will use digital technology for numerical experimentation and simulation.

TIME ALLOCATION

The Applied Mathematics course is designed for 180 hours of class contact time.

1 Word problems explicitly include all the information needed to solve them. The solver needs only to determine the appropriate model to use to compute the one correct answer.

Modelling problems require the solver to research the situation themselves, make reasonable assumptions, decide which variables will affect the solution, and develop a model that provides *a* solution that best describes the situation.

Key skills

In senior cycle, there are five key skills (Figure 5) identified as central to teaching and learning across the curriculum: information processing; being personally effective; communicating; critical and creative thinking; and working with others.

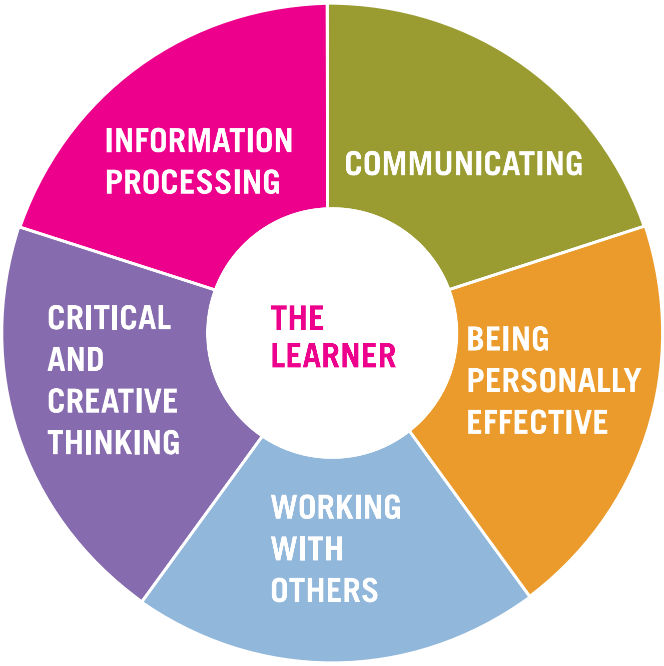


Figure 5: Key skills of senior cycle

The key skills are embedded within the learning outcomes of Applied Mathematics and are assessed in the context of those learning outcomes. This set of key skills, and the learning outcomes associated with them, became the NCCA Key Skills Framework (NCCA, 2009). The Key Skills Framework was developed to provide a common, unified approach for embedding the key skills across all future Leaving Certificate specifications. These skills are identified as being important for all students to achieve to the best of their ability, both during their time in school and in the future, and to fully participate in society, in family and community life, the world of work and lifelong learning. Applied Mathematics develops these skills in the following ways:

INFORMATION PROCESSING

Applying mathematics to real-world problems promotes independent research activities in which students are required to access a wide variety of external materials to analyse questions. The selection, evaluation, and recording of information are addressed; as students engage in the problem-solving cycle, they make decisions and judgments based on data and qualitative and quantitative information.

In particular, the problem-solving cycle requires careful assimilation and organising of the problem features before embarking on the calculation stage. In this information-intensive environment, students develop an appreciation of the differences between information and knowledge, and the roles that both play in making decisions and judgements.

CRITICAL AND CREATIVE THINKING

Modelling requires careful analysis of patterns and relationships, which develops skills of higher-order reasoning and problem solving. Part of the computational thinking involved in modelling is the ability to identify, analyse and deconstruct problems, explore options and alternatives, and hence solve problems. Hypothesising, making predictions, examining evidence, and reaching conclusions underpin the core of all the activities proposed in Applied Mathematics. As they develop these skills, students reflect critically on the forms of thinking and values that shape their own perceptions, opinions, and knowledge. This brings in the metacognitive dimension of knowledge which is essential in developing critical thought.

COMMUNICATING

Effective communication skills are developed through collaborative project work. Students communicate face-to-face and through digital media. Although literacy skills are not targeted directly, they are required by students to participate fully in the learning experience.

Online research and the use of external models requires and builds analysis and interpretation skills. Students need to read a wide range of information sources. Students are required to express and share their opinions and to hypothesise the reason clearly; debate and argument ensues which encourages engaging in dialogue, listening attentively and eliciting opinions, views and emotions.

There are opportunities to develop communication skills further as students compose and present using a variety of media.

WORKING WITH OTHERS

Applied Mathematics is underpinned by collaboration and working with others. Students gain some appreciation of group dynamics and the social skills needed to engage in collaborative work. This contributes to an appreciation that working collectively can help motivation, release energy, and capitalise on all the talents in a group.

One of the most beneficial outcomes of working with others is in identifying, evaluating and achieving collective goals. Students learn to negotiate and resolve differences of opinion as they discuss their different strategies and achieve compromise.

BEING PERSONALLY EFFECTIVE

This key skill contributes to the personal growth of students: they become more self-aware and use this awareness to develop personal goals. An important dimension of this key skill is in building the know-how of students to recognise how to get things done, how to garner and use resources effectively, and how to act autonomously. There is more than one way to answer a problem or set up a problem-solving strategy; there is no golden key to the answer. Students must develop confidence in their self-direction and exhibit tenacity and rigour. To be personally effective, students must build on the metacognitive dimension of knowledge, whereby they develop strategies to learn and to build on previous knowledge.

TEACHING AND LEARNING

Senior cycle students are encouraged to develop the knowledge, skills, attitudes and values that will enable them to become more independent in their learning and to develop a lifelong commitment to improving their learning.

Applied Mathematics supports the use of a wide range of teaching and learning approaches. The course is experiential in its structure and emphasises the practical application of mathematical knowledge to the world around us. As students progress, they will develop problem-solving skills that are transferable across different tasks and different disciplines, enabling them to see the power of mathematics.

By engaging in well-structured group discussions, students will develop skills in reasoned argument, listening to each other and reflecting on their own work and that of others.

Modelling is best learned by doing, and in planning for teaching and learning, teachers should provide ample opportunity for students to engage with the modelling process.

Engaging with real problems is motivating for students; it allows them to see the relevance of mathematics to situations that are important in their lives. The open-ended nature of mathematical modelling problems allows students to employ the mathematical tools that they prefer as well as practise skills they need to reinforce. The fact that the process itself involves iteration (evaluation and reworking of the model) clearly communicates that a straight path to success is unlikely and contributes to the development of students' tenacity.

The variety of activities that students engage in will enable them to take charge of their own learning by setting goals, developing action plans, and receiving and responding to assessment feedback. As well as varied teaching strategies, varied assessment strategies will support learning and provide information that can be used as feedback so that teaching and learning activities can be modified in ways that best suit individual students. By setting appropriate and engaging tasks, asking higher-order questions and giving feedback that promotes learner autonomy, assessment will support learning as well as summarising achievement.

Differentiation

The Leaving Certificate Applied Mathematics specification is differentiated to cater for students of differing abilities and levels of achievement.

DIFFERENTIATION THROUGH THE LEARNING OUTCOMES

Learning outcomes should be achievable relative to each student's ability level. Learning outcomes promote teaching and learning processes that develop students' knowledge and understanding incrementally, enabling them to analyse, evaluate and apply knowledge to different situations as they progress. Although students studying Ordinary level or Higher level can both critically analyse; the context, information and results associated with that analysis are presented at different levels.

ORDINARY LEVEL	HIGHER LEVEL
Only the learning outcomes that are presented in normal type. Students engage with a broad range of knowledge, mainly concrete in nature, but with some elements of abstraction or theory. They will be expected to demonstrate and use a moderate range of cognitive skills and mathematical tools to represent, analyse, make predictions and provide insights into real-world phenomena. They will be expected to select from a range of mathematical models and apply known solutions to a variety of problems in both familiar and unfamiliar contexts. They will understand the modelling cycle and use it to complete a modelling project.	All learning outcomes including those **in bold type.** Students engage with a broad range of knowledge, including theoretical concepts and abstract thinking with significant depth in some areas. They will be expected to demonstrate and use a broad range of cognitive skills and mathematical tools to represent, analyse, make predictions and provide insights into real-world phenomena. They will be expected to select from a range of mathematical models and build mathematical models to determine solutions to a variety of problems in both familiar and unfamiliar contexts. They will understand the modelling cycle and use it to complete a modelling project.

Figure 6: Differentiation through the learning outcomes in Leaving Certificate Applied Mathematics.

DIFFERENTIATION IN TEACHING AND LEARNING

Applied Mathematics provides numerous opportunities for teachers to teach the subject and select materials that meet the needs and interests of all students. The focus on the experiential approach to teaching and learning, which is central to applied mathematics, means that students can be engaged in learning activities that complement their own needs and ways of learning. The content matter of the course is specified in broad terms to allow the selection and exploration of topics in ways that are of most interest and relevance to the students.

Students vary in the amount and type of support they need to be successful. Levels of demand in any learning activity will differ as students bring different ideas and levels of understanding to it. The use of strategies for differentiated learning such as adjusting the level of skills required, varying the amount and the nature of teacher intervention, and varying the pace and sequence of learning will allow students to interact at their own level.

DIFFERENTIATION IN ASSESSMENT

Assessment of Leaving Certificate Applied Mathematics will be based on the learning outcomes in the specification. In the written assessment, Applied Mathematics will be assessed at two levels, Higher and Ordinary. At Higher level, the learning outcomes will be assessed at the highest skill level as demonstrated in the action verb of the learning outcome. At Ordinary level, the learning outcomes will be assessed at a moderate skill level, focusing on the demonstration of a basic understanding of facts and concepts and the application of concepts and theories. Examination questions will require students to demonstrate knowledge, understanding, application, analysis and evaluation appropriate to each level. Differentiation at the point of assessment will also be achieved through the language register of the questions asked, the stimulus material used, and the extent of the structured support provided for examination students at different levels.

In the second assessment component, the modelling project, the task will be common for Higher level and for Ordinary level. A differentiated marking scheme will apply. Students will be expected to follow the modelling cycle to use mathematics to represent, analyse, make predictions or provide insights into a real-world phenomenon and produce a written report.

4 Strands and learning outcomes

Strand 1: Mathematical modelling

In this unifying strand students learn about mathematical modelling as a process that uses mathematics to represent, analyse, make predictions or otherwise provide insight into real-world phenomena. The process is iterative and translates between the real world and mathematics in both directions and involves a number of stages. As they model authentic problems, students learn to appreciate the importance of mathematics in understanding the world around them and realise that although mathematical models are not perfect predictors of what will happen in the real world, they can offer important insights into key elements of a problem. Students become comfortable with uncertainty; not knowing an answer immediately does not deter them and learning from their peers is a valuable part of the process. They learn the importance of assumptions to the modelling process, and how they affect the validity of a model. They recognise that mathematical models are used to inform many decisions that directly affect their lives, and that being able to critically evaluate mathematical models is a desirable skill for them to acquire.

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
The problem-solving cycle	‣ describe a systematic process for solving problems and making decisions
Formulating problems	‣ research the background to a problem to analyse factors or variables that affect the situation ‣ determine information relevant to the problem ‣ decompose problems into manageable parts ‣ determine what assumptions are necessary to simplify the problem situation
Translating problems into mathematics	‣ use abstraction to describe systems and to explain the relationship between wholes and parts ‣ abstract the knowledge needed to build a mathematical model ‣ translate the information given in the problem together with the assumptions into a mathematical model that can be solved
Computing solutions	‣ compute a solution using appropriate mathematics ‣ create a mathematical model that can be interpreted by a computer ‣ use computational technology to solve problems ‣ solve the mathematical problem stated in the model ‣ analyse and perform operations in the model ‣ interpret the mathematical solution in terms of the original situation
Evaluating solutions	‣ refine a model and use it to predict a better solution to the problem; iterate the process ‣ communicate solution processes in a written report

Strand 2: Mathematical modelling with networks and graphs

In this strand, students learn about networks or graphs as mathematical models which can be used to investigate a wide range of real-world problems. They learn about graphs and adjacency matrices and how useful these are in solving problems. They are given further opportunity to consolidate their understanding that mathematical ideas can be represented in multiple ways. They are introduced to dynamic programming as a quantitative analysis technique used to solve large, complex problems that involve the need to make a sequence of decisions. As they progress in their understanding they will explore and appreciate the use of algorithms in problem-solving as well as consider some of the wider issues involved with the use of such techniques.

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
Networks and their associated terminology Matrices, matrix algebra and adjacency	‣ represent real-world situations in the form of a network ‣ use and apply the following network terminology: vertex / node, edge/arc, weight, path, cycle ‣ distinguish between connected and disconnected graphs, and between directed and undirected graphs ‣ represent a graph using an adjacency matrix, and reconstruct a graph from its adjacency matrix ‣ perform multiplication of square matrices by hand, with the help of a computer for larger matrices ‣ interpret the product of adjacency matrices ‣ translate between multiple representations of mathematical ideas
Minimum spanning trees applied to problems involving optimising networks and algorithms associated with finding these (Kruskal, Prim)	‣ demonstrate an understanding of the concepts of tree, spanning tree, minimum spanning tree in appropriate contexts ‣ use appropriate algorithms to find minimum spanning trees
Dynamic Programming and shortest paths as applied to multi-stage authentic problems such as ‣ stock control ‣ routing problems ‣ allocation of resources ‣ equipment replacement and maintenance	‣ use and apply dynamic programming terminology, such as stage, state, optimal policy ‣ apply Bellman's Principle of Optimality to find the shortest paths in a weighted directed acyclic network, and to be able to formulate the process algebraically ‣ apply Dijkstra's algorithm to find the shortest paths in a weighted undirected and directed network ‣ evaluate different techniques for solving shortest-route problems
Algorithms: ‣ Dijkstra ‣ Bellman ‣ Kruskal, Prim	‣ use algorithms to solve problems ‣ **distinguish between those algorithms which are greedy and those which use dynamic programming** ‣ **justify the use of algorithms in terms of correctness and their ability to yield an optimal solution**
Analysis of project scheduling networks Project scheduling and its associated terminology	‣ apply network concepts to project scheduling ‣ apply the concepts of critical path, early times, late times and floats to project scheduling

Strand 3: Mathematically modelling the physical world; kinematics and dynamics

In this unit students explore the motion of a particle and how characteristics of this motion can be described by functions. They are introduced to displacement, velocity, acceleration and force as vector quantities and recognise vectors as useful and established representations of quantitative data in dynamical systems. They make connections with their learning in Strand 5 LC Mathematics and interpret the derivative as a tool for finding instantaneous rates of change as well as consolidating their understanding of how calculus can be used to analyse motion. They learn how calculus was used by Newton to model dynamical change and use Newton's Laws together with other models[2] to solve problems involving accelerated particle motion in 1 and 2D. They understand Newton's Laws as mathematical models with underlying assumptions and as such, they break down under certain conditions. They understand conservation laws as fundamental principles in dynamics and use these to solve problems.

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
Kinematics: Particle motion in one direction. ‣ Common idealisations used to simplify authentic situations involving motion in 1D. ‣ Five basic concepts for describing the motion of a particle in 1D; position, displacement, velocity, acceleration and time. ‣ Graphical Representation and interpretation: displacement -time graphs, velocity-time graphs ‣ The kinematics formulae under constant acceleration • $v = u + a t$ • $s = ut + ½a t^2$ • $v^2 = u^2 + 2as$	‣ describe the motion of a particle in 1D [In a straight line] using words, diagrams, numbers, graphs and equations
Displacement as *a change of position* Velocity as the rate of change of displacement Acceleration as the rate of change of velocity ‣ Differentiation • The chain rule as a technique for differentiating composite functions ‣ The fundamental theorem of calculus ‣ The definite integral as the area under the curve (Riemann integral)	‣ interpret velocity and acceleration as derivatives ‣ transform the function describing one quantity (displacement, velocity, acceleration) into functions describing the other two quantities using algebra and/or calculus ‣ solve kinematics problems involving particle motion in one dimension ‣ derive the kinematic formulae of motion using calculus

2 Inextensible strings, Smooth rigid planes, Hooke's Law, Liquid and Aerodynamic Drag, Gravitational Force

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
Particle motion in 2D ▸ Elementary vector Algebra • addition – adding 2 successive displacements or velocity vectors • scalar multiplication • dot-product ▸ Elementary vector calculus ▸ Projectile Motion; time of flight, maximum height, maximum range, horizontal planes	▸ represent displacement as a vector ▸ apply and interpret vector algebra ▸ represent vectors in terms of components along unit vectors in 2 fixed orthogonal directions and in polar form ▸ calculate and interpret the dot product of vectors ▸ transform the function describing one quantity (displacement, velocity, acceleration represented as vectors) into functions describing the other two quantities using algebra and/or calculus ▸ represent and apply Newton's laws in vector form ▸ solve constant acceleration projectile motion problems involving displacement, velocity and time
Forces acting on a particle ▸ Mass ▸ Types of Forces; • Applied, Normal Reaction, Frictional, Resistant, Tension • Gravitational • Resultant ▸ Vectors as representations of quantitative data ▸ The concept of momentum ▸ Newton's Laws of motion ▸ Impulse ▸ Conservation of momentum for a two-particle system in one and two dimensions. ▸ Newton's experimental laws for collisions ▸ Elastic and inelastic collisions. Coefficient of restitution. ▸ Connected masses ▸ Rough and smooth surfaces ▸ Coefficient of friction ▸ Drag: Liquid, aerodynamic where $F \propto v^n$ ▸ Integration • by parts derived from the product rule • by substitution derived from the chain rule	▸ define a force as a measurable quantity ▸ draw free-body force diagrams for a particle on a smooth rigid fixed horizontal or inclined plane ▸ find the resultant force along a plane or inclined plane ▸ resolve forces on rough and smooth surfaces ▸ solve dynamic problems involving the motion of a particle under a constant resultant force ▸ solve dynamic problems involving particles that collide directly **and obliquely** ▸ solve dynamic problems involving connected masses ▸ describe the motion of a particle on a smooth/ rough, horizontal or inclined plane under a constant resultant force ▸ solve dynamic problems on rough and smooth surfaces ▸ solve dynamic problems involving resistive forces that are proportional to v^n $n \in \mathbf{R}$

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
Work, energy, conservative forces, conservation of energy and momentum ▸ A work-energy model for analysing particle motion ▸ The principle of conservation of energy ▸ Work done by a force • Gravitational force under the constant acceleration approximation • Variable conservative force • Force exerted on a particle by a linear stretched or compressed elastic spring and stretched strings	▸ define work done ▸ describe gravitational potential energy and how it relates to work done ▸ describe kinetic energy and how it relates to work done ▸ solve dynamic problems involving work done by a constant force ▸ solve dynamic problems involving the conservation of energy for variable conservative forces
Circular Motion of a particle	▸ solve problems involving the dynamics of a particle moving in a horizontal or vertical circle
Dimensional analysis	▸ evaluate and articulate whether an answer is reasonable by analysing the dimensions

Strand 4: Mathematically modelling a changing world

In this strand, students learn about difference and differential equations and how useful these are for modelling, simulating and understanding phenomena in the real world that involve change. Through exploration of authentic problems involving change they come to understand that differential equations are most useful for modelling situations where the change is continuous in time. If, however, the change is discrete in time, (e.g. happens incrementally) then students learn that difference equations are more appropriate models. Examples of incrementally changing phenomena that students encounter in this strand include biological applications such as population dynamics of species with discrete breeding seasons and applications related to finance and economics such as interest that is compounded monthly/annually, and supply and demand. Examples of continuously changing phenomena that students will encounter include motion involving variable force and acceleration, radioactive decay, cooling/warming, population growth, spread of disease, interest compounded continuously and issues such as deforestation, over-harvesting and climate change.

Students build on their exploration of sequences in Leaving Certificate Mathematics when they use recurrence relations to describe sequences. They develop an understanding of first and second -order difference equations as recursively defined sequences. Through analysing solutions in context, they consolidate their understanding that whilst mathematical models are not perfect predictors of what will happen in the real world, they can offer important insights and information about the nature and scope of a problem and can inform solutions. As they model with difference and differential equations students learn to abstract and apply techniques in a wide range of application areas.

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
Recurrence relations	‣ compute the first and higher differences of a given sequence of numbers
Real-world phenomena involving incremental change such as ‣ Malthusian growth, restricted growth ‣ interest/loan payment ‣ supply and demand ‣ spread of diseases such as measles	‣ identify real-world situations which can be suitably modelled by difference equations ‣ derive difference equations for real-world phenomena involving incremental change ‣ analyse, interpret and solve difference equations in context
Solving homogeneous and inhomogeneous difference equations numerically, graphically and with the aid of digital technology	‣ solve linear and non-linear difference equations
Analysing real-world phenomena involving continuous change Predictive nature of differential equations Techniques for solving differential equations • separation of variables • numerical solutions for digital implementation • graphical methods that provide qualitative information	‣ **identify real-world situations which can be suitably modelled by differential equations** ‣ **derive and interpret in context differential equations for real-world phenomena involving continuous change** ‣ **solve differential equations** • **first order separable** • **second order which can be reduced to first order** ‣ **interpret the solution of differential equations in context**

5 Assessment

Assessment in senior cycle involves gathering, interpreting and using information about the processes and outcomes of learning. It takes different forms and is used for a variety of purposes. It is used to determine the appropriate route for students through a differentiated curriculum, to identify specific areas of difficulty or strength for a given student and to test and certify achievement. Assessment supports and improves learning by helping students and teachers to identify next steps in the teaching and learning process.

As well as varied teaching strategies, varied assessment strategies will support learning and provide information that can be used as feedback so that teaching and learning activities can be modified in ways that best suit individual learners. By setting appropriate and engaging tasks, asking higher-order questions and giving feedback that promotes learner autonomy, assessment will support learning as well as summarising achievement.

Assessment for certification

Assessment for certification is based on the aim, objectives, and learning outcomes of this specification. Differentiation at the point of assessment is achieved through examinations at two levels–Ordinary level and Higher level.

There are two assessment components at each level: written examination and a modelling project.

Each component will be set and examined by the State Examinations Commission (SEC). Both components of assessment reflect the relationship between the application of skills and the theoretical content of the specification.

ASSESSMENT COMPONENT	WEIGHTING	LEVEL
Modelling project	20%	Higher and Ordinary
Written examination	80%	Higher and Ordinary

Figure 7: Overview of assessment.

MODELLING PROJECT

The modelling project assessment will require students to demonstrate proficiency in course content and skills that cannot be easily assessed by the written examination. The assessment will require students to present a solution to an authentic modelling problem, and to report on the work and process involved. Students must acknowledge (i.e. through citation, through attribution, by reference, and/or through acknowledgement in bibliographic entry) the source or author of all information or evidence taken from someone else's work. Student work will be submitted to and marked by the State Examinations Commission (SEC).

Through the modelling project, students are afforded an opportunity to engage in the full modelling cycle to propose a solution to an authentic problem in a real context. The modelling project will assess the student's ability to use mathematics to represent, analyse, make predictions or otherwise provide insight into a real-world phenomenon. The key skills of processing data and information, communicating, critical and creative thinking, being personally effective and working with others can be developed through all the learning in this course, and these skills will be applied through the student's engagement in the modelling project.

The modelling project will be based on a brief issued annually by the State Examinations Commission (SEC). A common brief will be issued for Ordinary level and Higher level. The brief will outline a modelling problem in a real-world scenario. There is no pre-determined solution strategy and the students have ownership of all decisions they make as they progress through the modelling cycle to arrive at their solution. The brief will also outline the parameters for the problem and for the format of the report, which will be submitted to the SEC for assessment. The modelling project will be completed in sixth year.

The modelling project requires students to demonstrate that they can:

- define a problem
- translate the problem to mathematics
- compute a solution
- analyse the solution and iterate the process.

The report must be the student's own work. Authentication procedures will be in place to ensure compliance with this requirement. These will include a protocol in relation to the use of internet-sourced material.

MODELLING PROJECT ASSESSMENT CRITERIA

THE STUDENT DEMONSTRATING A HIGH LEVEL OF ACHIEVEMENT:	THE STUDENT DEMONSTRATING A MODERATE LEVEL OF ACHIEVEMENT:	THE STUDENT DEMONSTRATING A LOW LEVEL OF ACHIEVEMENT:
states the problem statement concisely, early in the written report. References sources from background research. identifies several variables affecting the model and notes and justifies the need for the main factor that influences the phenomena being modelled. clearly identifies and justifies the assumptions used to develop the model and, where appropriate, states the limitations of the simplification of the problem due to the assumptions made. indicates exactly what the output of the model will be and, if appropriate, identifies the audience and/or perspective of the modeller.	identifies a problem statement which is not precise or consistent with other statements in the report. lists important parameters and variables properly, but without sufficient explanation. notes primary assumptions, but without justification.	presents a problem statement that is difficult to understand or is buried in the text. identifies assumptions and justifies them, but they are difficult to identify in the text. barely mentions variables/ parameters or, if mentioned, they are difficult for the reader to identify in the text.
provides clear insight with logical mathematical reasoning into the mathematical method(s) used to describe the relationship between the variables, and to solve the problem. Presents a plausible approach and outcome.	states a mathematical approach, however with aspects of the method(s) which are inconsistent, difficult to understand or incomplete.	states a model which contains fixable mathematical errors.
clearly presents an accurately-computed solution and analysis of the relationship between variables, supported where appropriate with visual aids and graphic representation that is consistent with the original problem statement.	states an answer, however with aspects of the solution(s) which are inconsistent, difficult to understand or incomplete (e.g. fails to identify units of measure).	states an answer but without contextual background (i.e. appropriate graphics, appropriate units, etc.). arrives at no solution.
addresses the viability and reliability of the mathematical modelling solution. considers how sensitive the model is to changes in parameter values or altered assumptions; how it compares to other solutions or historical data. The model is refined and the process iterated.	addresses the viability and reliability of the mathematical modelling solution, however with analysis which lacks proper dimensionality, e.g. obvious consequences of the stated outcome are ignored or well-known comparisons are disregarded.	provides some analysis but without any sense of perspective. uses incorrect mathematics in the analysis.
presents a paper that is well-formatted and enjoyable to read, with easy to interpret visual aids (if appropriate).	presents a paper with multiple spelling, formatting or grammatical errors, visual aids which are missing key readability features or which do not clearly connect to the solution.	presents a paper with significant disregard for common spelling, grammatical and mathematical rules.

Figure 8: Modelling project assessment criteria.

Written examination

The written examination will consist of a range of question types. The questions will require students to demonstrate knowledge and understanding, and an ability to apply, analyse and evaluate appropriate to each level. The key skills are embedded in the learning outcomes and will be assessed in the context of the learning outcomes. The written examination paper will assess, appropriate to each level:

- knowledge and understanding of mathematics from the three contextual strands of the specification
- application of mathematics to represent, analyse, make predictions and/or provide insights into authentic real-world phenomena
- critical thinking, the ability to analyse and evaluate information and to form reasonable and logical argument based on evidence.

The written examination will be two and a half hours in duration and will include a selection of questions that allow for:

- the knowledge and understanding of the student to be assessed in relation to the three contextual strands of study
- the student to display his/her ability to use mathematics to represent, analyse, make predictions and/or provide insights into authentic real-world phenomena
- the student to display his/her skills of critical thinking, analysis, evaluation and his/her application of mathematical knowledge and understanding in real contexts.

WRITTEN EXAMINATION ASSESSMENT CRITERIA

A high level of achievement in the written examination is characterised by a thorough knowledge and understanding of mathematical terms, concepts and theories from across the whole specification with few significant omissions. Students consistently apply their knowledge and understanding of mathematics from the three contextual strands to both familiar and new contexts, where appropriate. They accurately use mathematics to represent real-world phenomena, analyse and evaluate information and data from different sources; manipulation of data, where appropriate, will be correct. Students present logical arguments and ideas which are clearly based on evidence.

A moderate level of achievement in the written examination is characterised by a good knowledge and understanding of mathematical terms, concepts and theories from many parts of the specification. Students apply their knowledge and understanding of mathematics from the three contextual strands to familiar contexts and in some new contexts, where appropriate. They adequately use mathematics to represent and analyse real-world phenomena. They carry out adequate levels of analysis and evaluation on information and data from different sources; much of their manipulation of data, where appropriate, will be correct. Students present arguments and ideas which, in the main, are based on evidence.

A low level of achievement in the written examination is characterised by a limited knowledge and understanding of mathematical terms, concepts and theories. Students select appropriate facts, concepts and theories to apply to familiar contexts. They use basic mathematics to represent and analyse real-world phenomena. They carry out basic levels of analysis and evaluation of information and data and carry out basic manipulation of data, where appropriate. Students present explanations which are referenced to some evidence.

Leaving Certificate grading

Leaving Certificate Applied Mathematics will be graded using an 8-point grading scale. The highest grade is a Grade 1; the lowest grade is a Grade 8. The highest seven grades (1-7) divide the marks range 100% to 30% into seven equal grade bands 10% wide, with a grade 8 being awarded for percentage marks of less than 30%. The grades at Higher level and Ordinary level are distinguished by prefixing the grade with H or O respectively, giving H1-H8 at Higher level, and O1-O8 at Ordinary level.

GRADE	% MARKS
H1/O1	90-100
H2/O2	80<90
H3/O3	70<80
H4/O4	60<70
H5/O5	50<60
H6/O6	40<50
H7/O7	30<40
H8/O8	<30

Figure 9: Leaving Certificate grading scale.

REASONABLE ACCOMMODATIONS/INCLUSION

This Leaving Certificate Applied Mathematics specification requires that students engage with mathematical modelling on an ongoing basis throughout the course. In addition, the assessment involves a coursework element, which accounts for 20% of the total marks awarded. In this context, the scheme of Reasonable Accommodations, operated by the State Examinations Commission, is designed to assist candidates in the Leaving Certificate who have physical/medical/sensory and/or specific learning difficulties.

Reasonable accommodations are designed to remove as far as possible the impact of a disability on a candidate's performance, so that he or she can demonstrate in an examination his or her level of achievement—they are not designed to compensate for a possible lack of achievement arising from a disability.

Applications for reasonable accommodations are considered within a published framework of principles (Expert Advisory Group Report, January 2000) and are submitted by the school which a candidate attends on prescribed application forms. Applications are normally invited one year in advance of the examination concerned.

NCCA
An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment

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